



## Friday 23 June 2017 - Morning

## **A2 GCE MATHEMATICS (MEI)**

4754/01A Applications of Advanced Mathematics (C4) Paper A

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### **OCR** supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

#### **INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail
  of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of 16 pages. The Question Paper consists of 4 pages.
   Any blank pages are indicated.
- This paper will be followed by Paper B: Comprehension.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

#### Section A (36 marks)

- 1 (i) Express  $\frac{5-x}{(2-x)(1+x)}$  in partial fractions. [3]
  - (ii) Hence or otherwise find the first 3 terms of the binomial expansion of  $\frac{5-x}{(2-x)(1+x)}$  in ascending powers of x. [5]
- 2 The equation of a line is  $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$  and the equation of a plane is 3x + 4y z = 17.
  - (i) Find the coordinates of the point of intersection of the line and the plane. [4]
  - (ii) Find the acute angle between the line and the normal to the plane. [4]
- 3 Fig. 3 shows the curve  $y = \sqrt{1 + e^{2x}}$ .

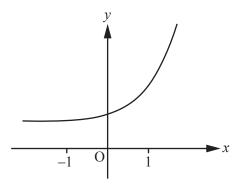


Fig. 3

The value of  $\int_{-1}^{1} \sqrt{1 + e^{2x}} dx$  is to be estimated using the trapezium rule.  $T_2$  and  $T_4$  are the estimates obtained from the trapezium rule using 2 strips and 4 strips respectively.

- (i) Explain whether  $T_4$  is greater or less than  $T_2$ . [2]
- (ii) Evaluate  $T_4$ , giving your answer to 3 significant figures. [4]
- 4 Vectors  $\mathbf{u}$  and  $\mathbf{v}$  are given by  $\mathbf{u} = \mathbf{i} 7\mathbf{j} 2\mathbf{k}$  and  $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + 5\mathbf{k}$ , where a and b are constants.

Find a and b given that the magnitude of  $\mathbf{v}$  is  $\sqrt{27}$  and that  $\mathbf{u}$  and  $\mathbf{v}$  are perpendicular. [6]

- 5 Solve the equation  $4 \tan \theta \tan 2\theta = 1$ , for  $0^{\circ} < \theta < 180^{\circ}$ .
- 6 The number of bacteria in a population at time *t* is denoted by *P*. The rate of increase of *P* is proportional to the square root of *P*.
  - (i) Write down a differential equation relating P, the time t, and a constant of proportionality k. [1]
  - (ii) Verify that  $P = (A + Bt)^2$ , where A and B are constants, satisfies the differential equation, and find k in terms of B.

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#### Section B (36 marks)

7 The curve shown in Fig. 7 passes through the origin and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9x}{4(y+3)} \,.$$

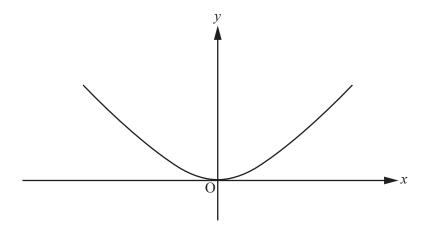


Fig. 7

(i) Show by integration that the equation of the curve is  $9x^2 - 4y^2 - 24y = 0$ . [5]

The finite region bounded by the curve and the line y = 2 is rotated through 180° about the y-axis.

(ii) Find the volume of the solid of revolution generated, giving your answer as an exact multiple of  $\pi$ .

[4]

(iii) Use the substitutions  $x = 2 \tan \theta$  and  $y = 3(\sec \theta - 1)$ 

- (A) to verify that  $9x^2 4y^2 24y = 0$ ,
- (B) to show that  $\frac{9x}{4(y+3)}$  can be expressed as  $k \sin \theta$ , where k is a constant to be found.

Hence find the exact gradient of the curve at the point with x-coordinate 2. [9]

**8** Fig. 8 shows the curve with parametric equations

$$x = \cos 2\theta$$
,  $y = \cos \theta + 2\sin \theta$ , for  $-\pi < \theta \le \pi$ .

The curve intersects the x-axis at A, and the points B and C have maximum x- and y-coordinates respectively.

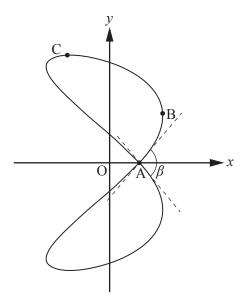


Fig. 8

- (i) Find the value of  $\theta$  corresponding to the point B. Hence find the coordinates of the point B. [3]
- (ii) Express  $\cos \theta + 2 \sin \theta$  in the form  $R \cos(\theta \alpha)$ , where R > 0 and  $0 < \alpha < \frac{1}{2}\pi$ . [5]
- (iii) Hence find the coordinates of the points A and C. [5]

The angle  $\beta$  is the angle between the tangents to the curve at A.

(iv) Find  $\frac{dy}{dx}$  in terms of  $\theta$ . Hence, assuming the scales of the x- and y-axes are equal, find  $\beta$ , giving the answer in radians correct to 2 decimal places. [You may assume the curve is symmetrical about the x-axis.]

#### **END OF QUESTION PAPER**



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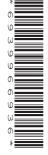


## Friday 23 June 2017 - Morning

## **A2 GCE MATHEMATICS (MEI)**

**4754/01** Applications of Advanced Mathematics (C4)

**INSTRUCTIONS** 



The examination is in two parts:

Paper A (1 hour 30 minutes) Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B is not issued until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding Paper A script. For Paper A only the candidates' Printed Answer Books, in the same order as the attendance register, should be sent for marking; the Question Paper should be retained in the centre or recycled. For Paper B only the Question Papers, on which the candidates have written their answers, should be sent for marking; the Insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the invigilator's desk at all times during the morning of Friday 23 June 2017.



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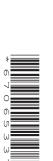
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## Friday 23 June 2017 - Morning

### **A2 GCE MATHEMATICS (MEI)**

4754/01A Applications of Advanced Mathematics (C4) Paper A

#### PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

#### **OCR** supplied materials:

- Question Paper 4754/01A (inserted)
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



Candidate forename				Candidate surname			
Centre number				Candidate nu	umber		

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## Section A (36 marks)

1 (i)	
1 (ii)	

2 (i)	
, ,	
2 (ii)	
<b>2</b> (11)	

3 (i)	
3 (ii)	

4	
_	

5	

(4)	
6 (i)	
6.000	
6 (ii)	

## Section B (36 marks)

7(i)	

7 (ii)	

) (	(Answer the whole of Q7(iii) on this page.)
L	
_	
_	
L	

8 (i)	

8 (ii)	

8(iii)	

8 (iv)	

## ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

I .	



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## **GCE**

# **Mathematics (MEI)**

Unit 4754A: Applications of Advanced Mathematics: Paper A

Advanced GCE

Mark Scheme for June 2017

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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### **Annotations and abbreviations**

Annotation in scoris	Meaning
✓ and X	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0 M1	Method mark awarded 0, 1
A0 A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

### Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

#### М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

#### Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Qι	iestion	Answer	Marks	Guidance
1 (i	)	$\frac{5-x}{(2-x)(1+x)} = \frac{A}{2-x} + \frac{B}{1+x}$ $\Rightarrow 5-x = A(1+x) + B(2-x)$ $x = 2 \Rightarrow 3 = 3A, A = 1$	M1 A1	Cover up, substitution or equating coefficients
		$x = -1 \Rightarrow 6 = 3B \Rightarrow B = 2$	A1 [3]	
(i	i)	$\frac{A}{2-x} = \frac{A}{2} (1 - \frac{1}{2}x)^{-1}$	B1	Or equivalent
		$= \frac{A}{2}(1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2!}(-\frac{1}{2}x)^2 + \dots)$	M1	All three correct <b>unsimplified</b> binomial coefficients (not nCr) soi <b>for either expansion</b> i.e. 1, $-1$ and $\frac{(-1)(-2)}{2}$ . Or correct simplified coefficients seen
		$= A \left( \frac{1}{2} + \frac{1}{4}x + \frac{1}{8}x^2 + \dots \right)$	A1ft	Ignore any subsequent incorrect terms – ft their A from (i) only
		$\frac{B}{1+x} = B(1+x)^{-1} = B(1-x+x^2+)$	A1ft	Ignore any subsequent incorrect terms – ft their <i>B</i> from (i) only
		$\frac{5-x}{(2-x)(1+x)} = \frac{5}{2} - \frac{7}{4}x + \frac{17}{8}x^2 + \dots$	A1	www cao – ignore any higher order terms stated – isw after correct expansion seen
2 (			[5]	
2 (i	)	$x = -1 + 2\lambda, y = -3\lambda, z = 1 + \lambda$	M1*	Condone a single sign slip – maybe implied by later working
		$\Rightarrow 3(-1+2\lambda)+4(-3\lambda)-(1+\lambda)=17$	M1dep*	Substituting their <i>x</i> , <i>y</i> and <i>z</i> into the correct plane equation (condone a further single slip if intention clear)
		$\Rightarrow -7\lambda = 21, \lambda = -3$	A1	www cao
		point of intersection is therefore $(-7, 9, -2)$	A1	www cao – condone $x = -7$ , $y = 9$ , $z = -2$ – condone answer given as a column vector
			[4]	

Question	Answer	Marks	Guidance
(ii)	$\mathbf{u} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k},  \mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	B1	Identifying and using vectors <b>u</b> and <b>v</b> and no other vectors
	$\cos \theta = \frac{3 \times 2 + 4 \times (-3) + (-1) \times 1}{\sqrt{26}\sqrt{14}}$	M1	Correct formula (including cosine), with correct substitution of their <b>u</b> and their <b>v</b> – with at least one vector correct. Condone either a single numerical slip <b>or</b> a single sign slip (but not one of each) if intention is clear. It must be clear where the slip comes from e.g. if the magnitude of one vector is stated incorrectly with no working then M0
	$\theta = 111.5^{\circ}$	A1	Allow 111 or 112 or 1.94 or 1.95 (must come from correct working) –
			may be implied by the correct answer of 68.5
	so acute angle between line and normal is 68.5°	A1	www cao (or 1.20 in radians) or better (68.47546, 1.195122) A0 if choice of answers
			If sine used then it must be a complete method for finding the correct
			angle - so for the M mark need to see
		[4]	$\theta = \sin^{-1} \left( \frac{3 \times 2 + 4 \times (-3) + (-1) \times 1}{\sqrt{26} \sqrt{14}} \right) + 90^{\circ} \text{ (oe) if no attempt to add } 90 \text{ M0}$
3 (i)	$T_4 < T_2$	B1	oe (e.g. $T_4$ is less than $T_2$ )
	The approximation is an over-estimate, as the trapezia are above the curve therefore the error becomes less when the number of strips increases	B1	Must see mention of 'over-estimate' and 'above' and 'increasing strips'
		[2]	
(ii)	$0.5 \times 0.5$	B1	For using 0.25 oe
	$\{1.0655+2.8963+2(1.1695+1.4142+1.9282)\}$	M1	The M mark requires the correct {} bracket structure. It needs the
		A1	first bracket to contain the first y value plus the last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values with no additional values. Allow an error in one value <b>or</b> the omission of one value from the second bracket M0 if using x values. All values given to at least 3sf or exact
			The A mark is for the correct {} bracket with no errors (12.98 or
	$T_4 = 3.25$	A1	13.0 implies M1A1) cao (3.25 with no working scores 0/4) – must be given to 2dp only (for reference correct answer is 3.2465079) SC: bracketing error 0.25×(1.0655+2.8963)+2(1.1695+)
		[4]	scores B1M1A0A0 unless the final answer implies the correct calculation. An answer of 10.014 usually indicates this error

	Question	Answer	Marks	Guidance
	OR	0.5587+ 0.6459 + 0.8356+ 1.2061		Separate trapezia B1 – one area correct (implies 0.25)
				M1 – three correct (equivalent to one error)
				A1 – all four correct
				A1 – cao of 3.25
4		$ \mathbf{v}  = \sqrt{a^2 + b^2 + 25} = \sqrt{27}$	M1*	Condone one error on either lhs or rhs (but not both)
		$\Rightarrow a^2 + b^2 = 2$	A1	oe - allow unsimplified but square roots must be removed
		a - 7b - 10 = 0	B1	cao (oe) – must include = 0 (if all terms collected on one side)
		$a = 7b + 10 \Rightarrow (7b + 10)^2 + b^2 = 2$	M1dep*	Correct method for solving their simultaneous equations — which would result in a three-term quadratic equation (= $0$ maybe implied) in either $a$
				or $b$ – if correct the equation in $a$ is $49a^2 + (a-10)^2 = 98$
		$b = -\frac{7}{5}$	A1	www cao
		$a = \frac{1}{-}$	A1	www cao - award B1 for each correct answer if no working seen
		5	[6]	
5		$4\tan \theta \tan 2\theta = 1 \Rightarrow 4\tan \theta \cdot \frac{2\tan \theta}{1 - \tan^2 \theta} = 1$	M1*	Use of double angle formula for tan to get an equation in tan – allow one sign slip only
		$\Rightarrow 8\tan^2\theta = 1 - \tan^2\theta$		
		$\Rightarrow \tan^2 \theta = 1/9$	M1dep*	Re-arranges to $\tan^2 \theta = k$ where $k > 0$ or attempt to solve
				$a \tan^2 \theta - b = 0$ where $b/a > 0$
		$\tan \theta = 1/3 \text{ or } -1/3$		
		$\theta = 18.43^{\circ} \text{ or } 161.57^{\circ}$	A1	One correct answer to at least 1dp
		$\theta = 18.43^{\circ} \text{ and } 161.57^{\circ}$	A1	Both answers correct to at least 1dp
			[4]	SC A1A0 for answers which round to 0.322 <b>and</b> 2.82 (radians) Answers with no working can score B1 B1 (max 2/4) if correct Ignore additional solutions outside the range. If any additional solutions given inside the range of $0 < \theta < 180$ and full marks would have been awarded then remove last mark (so 3/4)

	Quest	ion	Answer	Marks	Guidance
6	(i)		$\frac{\mathrm{d}P}{\mathrm{d}t} = k\sqrt{P}$	B1 [1]	oe – condone $dP = k\sqrt{P}dt$
	(ii)		$P = (A + Bt)^2 \Rightarrow \frac{\mathrm{d}P}{\mathrm{d}t} = 2(A + Bt)B$	M1	Attempt at chain rule – allow this mark if $B$ absent or incorrect but must include $2(A+Bt)$ - if multiplied out condone one error in differentiation
			$\frac{\mathrm{d}P}{\mathrm{d}t} = 2B\sqrt{P}$	A1	oe - correctly showing that the rate of increase of $P$ is proportional to the square root of $P$
			k = 2B	A1 [3]	Not for $B = \frac{k}{2}$
		OR	$\int \frac{\mathrm{d}P}{\sqrt{p}} = k \int \mathrm{d}t \Rightarrow 2\sqrt{P} = kt(+c)$ $P = \left(\frac{1}{2}kt + A\right)^2$	M1	Separates their variables correctly and attempt to integrate for their differential equation given in (i) – for an attempt powers must increase by 1 (oe) but not for $k$ – condone lack of + $c$
			$P = \left(\frac{1}{2}kt + A\right)^2$	A1	Correct integration - accept any (correct) constant for $A$ - coefficient of $t$ maybe implicit stated e.g. $P = (Ct + A)^2$ with $k = 2C$ seen
			k = 2B	A1	Not for $B = \dots$
7	(i)		$k = 2B$ $\int 4(y+3)  \mathrm{d} y = \int 9x  \mathrm{d} x$	M1	Separating variables – if no subsequent work then integral signs needed, but allow omission of dx and/or dy but must be correctly placed if present – allow errors in re-arranging but x and y on separate sides
			$2y^2 + 12y = \dots$	A1	oe - Correct integration of y terms (or $2(y+3)^2$ oe)
			$\dots = \frac{9}{2}x^2(+c)$	A1	oe - Correct integration of $x$ terms – condone omission of + $c$ for these two A marks
			when $x = 0$ , $y = 0$ , $\Rightarrow 0 = 0 + c$ , so $c = 0$	B1	cao (previous three marks must have been awarded) – need to show that $c = 0$ . As a minimum $x = 0$ , $y = 0$ , $c = 0$
			$\Rightarrow 4y^2 + 24y = 9x^2$		
			$\Rightarrow 9x^2 - 4y^2 - 24y = 0$ *	A1	AG www (dependent on all previous marks in this part) – must show at least one step of intermediate working after integration – must have $= 0$
					SC: working backwards – B1 for $18x - 8y \frac{dy}{dx} - 24 \frac{dy}{dx} = 0$ and then B1 for
				[5]	re-arranging correctly to $\frac{dy}{dx} = \frac{9x}{4(y+3)}$ (so max. 2/5)

Question	Answer	Marks	Guidance
(ii)	$V = \pi \int_0^2 x^2  \mathrm{d} y$	M1	M1 for $k(\pi) \int_0^2 x^2(dy)$ with correct limits and $k = 1$ or $\frac{1}{2}$ , allow correct limits seen or implied later. If formula not stated then must substitute for their $x^2$ to imply this formula – condone lack of $\pi$ for the M mark and dy throughout (condone incorrect use of dx too)
	$= \pi \int_0^2 \frac{4}{9} (y^2 + 6y)  \mathrm{d} y$	A1	Correct (or with a factor of ½) – limits may be seen or implied through later working
	$= \pi \int_0^2 \frac{4}{9} (y^2 + 6y)  dy$ $\frac{4}{9} \left( \frac{y^3}{3} + 3y^2 \right)$ $= \frac{176}{27} \pi \text{ or } 6\frac{14}{27} \pi$	B1	or $\frac{2}{9} \left( \frac{y^3}{3} + 3y^2 \right)$ (but only if $k = \frac{1}{2}$ )
	$= \frac{176}{27} \pi \text{ or } 6\frac{14}{27} \pi$	A1 [4]	Exact (oe) – mark final answer (so no isw if correct answer is e.g. halved) but if exact value seen and is then followed by 20.4785 then isw

Quest	ion	Answer	Marks	Guidance
(iii)	(A)	= $9(2 \tan \theta)^2 - 4(3(\sec \theta - 1))^2 - 24(3(\sec \theta - 1))$	M1	Sub. for x and y – condone one slip e.g. using $3 \sec \theta - 1$
		$= 36 \tan^2 \theta - 36 \sec^2 \theta + 72 \sec \theta - 36 - 72 \sec \theta + 72$	A1	cao (multiplied out) – accept if = 0 and multiplied out correctly
		$= 36 \tan^2 \theta - 36 (1 + \tan^2 \theta) + 72 \sec \theta - 36 - 72 \sec \theta + 72$	M1	Use of correct formula $\sec^2 \theta = 1 + \tan^2 \theta$ (or $\sin^2 \theta + \cos^2 \theta = 1$ if converted to $\sin/\cos$ ) soi - maybe awarded before previous A mark – for those that set the lhs = 0 this mark can be awarded for a clear attempt to derive one of these identities
		=-36-36+72=0*	A1	AG www – all working must be explicitly shown including use of relevant Pythagorean identity. If candidates set their expression equal to zero then all four marks can be awarded provided all working is correct (so no dividing through by a constant before the expression has been put equal to zero) <b>and</b> a conclusion is required (so for those that derive a correct trig. identity then we must see a statement such as 'true' etc. accept as a minimum a tick. If following working from both sides of their equation $0 = 0$ seen then either a statement or re-stating the equation $9x^2 - 4y^2 - 24y = 0$ is required) – no $\theta$ missing in tan etc.
(iii)	(B)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{18\tan\theta}{12\sec\theta}$	B1	Maybe implied by later working
		$=\frac{18\frac{\sin\theta}{\cos\theta}}{12\frac{1}{\cos\theta}}$	M1	Correct use of both $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$ give bod if stated somewhere in their solution
		$=\frac{3}{2}\sin\theta$	A1	$k = \frac{3}{2}$ oe - need not be explicitly stated so accept $\frac{3}{2}\sin\theta$
		when $x = 2$ , $\theta = \pi/4$	B1	Must be exact – condone $\theta = 45^{\circ}$ or $y = 3(\sqrt{2} - 1)$
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}\sin\frac{\pi}{4} = \frac{3}{2\sqrt{2}}$	A1	
			[9]	correct final answer implies previous B mark

Question	Answer	Marks	Guidance
8 (i)	x = 1 $\theta = 0$ $y = \cos 0 + 2\sin 0 = 1$ , (so B is (1, 1))	B1 B1 B1 [3]	Condone if derived from $\theta = \pi$ Must be explicitly stated www (not from $2\theta = 2\pi$ ) - condone $x = 1$ and $y = 1$
(ii)	$\cos\theta + 2\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$	[e]	
	$\Rightarrow R\cos\alpha = 1, R\sin\alpha = 2$	M1 A1	Correct pairs. Condone sign errors only for the M mark – allow use of $\theta$ for $\alpha$ only if recovered later by stating a value for $\tan \alpha$ or $\alpha$ not $\theta$
	$R^2 = 1^2 + 2^2 = 5 \Rightarrow R = \sqrt{5}$	B1	Or 2.24 or better, not $\pm \sqrt{5}$ unless $+\sqrt{5}$ chosen
	$\tan \alpha = 2 \Rightarrow \alpha = 1.107$	M1 A1	For M1 ft their pairs (condone sign errors but division must be the correct way round). A1 for 1.107 or better (accept 1.11), with no errors seen in method for angle – A0 if in degrees SC: If candidates state that $\cos \alpha = 1$ , $\sin \alpha = 2 \Rightarrow \tan \alpha = 2$ this could score M0 A0 B1 M1 A1 (so max 3/5) Note that candidates who state the correct values of $R$ and $\alpha$ with no (wrong) working seen can score full marks
(iii)	At A, $\cos(\theta - \alpha) = 0$ , so $\theta - \alpha = \pi/2$ , $\theta = 2.678$ Or $\cos\theta + 2\sin\theta = 0 \Rightarrow 1 + 2\tan\theta = 0$ $\therefore \theta = -0.464$	B1ft	$\theta = \frac{\pi}{2} + \text{their } \alpha \ (\theta = 2.677945) \text{ or } -0.4636476 \text{ from tan - allow}$
	x = 0.6, y = 0	B1	in degrees $(-26.56505)$ www their $x$ value must be given exactly or correctly rounded to $0.6$ (condone trailing 0s) but do not accept $0.600087$ must also state $y = 0$
	At C, $cos(\theta - \alpha) = 1$ , so $\theta - \alpha = 0$ , $\theta = 1.107$	B1ft	$\theta$ = their $\alpha$ from (ii) (allow $\theta$ = 2.03444) - allow in degrees (63.4349) or from $-\sin\theta + 2\cos\theta = 0$ (condone if stated as dy/dx)
	x = -0.6	B1	www exact or correctly rounded to $-0.6$ (do not accept e.g. $-0.5997$ as their final answer)
	$y = \sqrt{5}$	B1ft [ <b>5</b> ]	Exact or 2.24 or better (2.2360679) or ft their <i>R</i>

Question	Answer	Marks	Guidance
(iv)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta - \sin\theta}{-2\sin 2\theta}$ when $\theta = 2.678$ , $\frac{dy}{dx} = -1.398$ or $\theta = -0.464$ , $\frac{dy}{dx} = 1.398$ $\tan \frac{1}{2}\beta = 1.398$	M1* A1 A1	M1 for their $(dy/d\theta)$ / their $(dx/d\theta)$ in terms of $\theta$ with at least one term correct (soi) cao (oe) – allow unsimplified even if subsequently cancelled incorrectly i.e. can isw  Allow $\pm$ 1.40 or better ( $\pm$ 1.3975424)
	angle between branches = 1.90 (radians)	A1 [5]	$\tan\left(\frac{1}{2}\beta\right)$ equal to either +/- their numerical gradient at A but must be using $\theta$ values and not $x$ values  1.90 or better (1.8994312) – not in degrees

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# 4754 Applications of Advanced Mathematics (C4)

#### **General Comments:**

On the whole candidates found Paper A this year slightly less demanding than last year and the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Questions 1, 2, 4, 7(ii) and 7(iii))
- Failure to include a constant of integration (Question 7(i))
- Failure to give clear descriptions in the comprehension paper (Questions 2 and 3)
- Inappropriate accuracy, for example in Question 3(ii), candidates either gave insufficient accuracy (answers to 1 decimal place) or they gave too much accuracy (answers to 3 or more decimal places). Candidates are reminded to give answers to 1 decimal place for questions involving trigonometry (Questions 2(ii) and 5)
- Failure to give exact answers when required (Question 7)
- Failure to give sufficient detail when verifying given results (Questions 6(ii), 7(i) and 7(iii))

Quite a number of candidates did not attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and are sent separately for marking.

#### **Comments on Individual Questions:**

Paper A

Question 1

Part (i) was answered extremely well with the vast majority of candidates correctly expressing  $\frac{5-x}{(2-x)(1+x)}$  in partial fractions.

In part (ii) most candidates used their answer to part (i) in their attempt to find the binomial expansion of  $\frac{5-x}{(2-x)(1+x)}$  although some candidates did (with varying degrees of success) attempt to expand  $(5-x)(2-x)^{-1}(1+x)^{-1}$  directly. Whilst the majority of candidates correctly dealt with the expansion of  $\frac{2}{1+x}$  (and so scored at least two marks in this part) it was surprising how many

candidates (at this level) struggled in re-writing  $\frac{1}{2-x}$  as  $\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}$ . In some cases it was clear

that candidates either did not realise or even recognise that the 2 inside the bracket had to be removed before this term could be binomially expanded. Those candidates who expanded both terms correctly usually went on to score full marks.

Question 2

Part (i) was answered extremely well with the vast majority of candidates correctly substituting the parametric form of the equation of the line into the equation of the plane to find the value of the parameter at the point of intersection (which if done correctly gave  $\lambda = -3$ ). This value was then substituted back into the equation of the line to find the coordinates of the point of intersection. Although many candidates gave their answer as a position vector (rather than as coordinates) this was condoned by examiners.

The main stumbling block in part (ii) was not the application of the scalar product to find the angle between the line and the normal to the plane but it was in the choosing of the correct two direction

vectors; in many cases candidates incorrectly used either 
$$\begin{pmatrix} -7\\9\\-2 \end{pmatrix}$$
, or even more surprisingly  $\begin{pmatrix} -1\\0\\1 \end{pmatrix}$ 

It was pleasing to note that in most cases, where candidates found the correct obtuse angle of 111.5°, they then correctly subtracted this value from 180° to obtain the correct acute angle.

#### Question 3

The first mark in part (i) was awarded to the vast majority of candidates for correctly stating that  $T_4$  was less than  $T_2$  although some candidates did not make it explicitly clear which value of the two was the least. Candidates found the second mark a lot harder to come by as it was not sufficient to simply state that the approximations given by the trapezium rule were an over-estimate. Candidates needed to make it clear that these approximations were an over-estimate because the tops of the trapezia are above the curve which would then, in turn, mean that the error (in using a trapezium rule approximation for the value of the integral) would become less when the number of strips increases.

Part (ii) was answered extremely well with the vast majority of candidates giving the correct answer of 3.25. When errors occurred it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the *x* values or to not give the answer to the required 3 significant figures.

### Question 4

On the whole, candidates made an impressive start to this unstructured vector question and the majority of candidates correctly achieved  $\sqrt{a^2+b^2+25}=\sqrt{27}$  and a-7b-10=0. It was therefore extremely disappointing that at this level so many candidates failed to solve this pair of simultaneous equations accurately. Too many candidates simplified  $\sqrt{a^2+b^2+25}=\sqrt{27}$  to a+b+5=27 or re-wrote a-7b-10=0 as  $a^2-49b^2-100=0$  and so therefore did not obtain a three-term quadratic equation in either a or b. Of those that did achieve a correct equation, most notably  $(7b+10)^2+b^2=2$ , it was disappointing how many then expanded this as

 $49b^2 + 100 + b^2 = 2$  and still arrived at a real solution for the value of b from the equation  $b^2 = -\frac{49}{25}$ .

For those candidates who did expand correctly most achieved the correct values for both a and b.

#### Question 5

It was pleasing to note that most candidates used the correct double angle formulae for  $\tan 2\theta$  to obtain a correct equation in terms of  $\tan \theta$ . However, some candidates over complicated the problem by re-writing tan in terms of sin and cos and in these cases it was extremely rare for candidates to make any real significant progress. Of those that correctly re-arranged

$$4\tan\theta \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=1$$
 to  $\tan^2\theta=\frac{1}{9}$  it was disappointing that so many candidates then only considered the solutions of the equation  $\tan\theta=\frac{1}{3}$  and ignored any possible solutions that would have come from the equation  $\tan\theta=-\frac{1}{3}$ .

#### Question 6

In part (i) most candidates correctly wrote down the differential equation relating P, the time t, and the constant k. The most common errors in this part were those candidates who wrote  $\frac{\mathrm{d}t}{\mathrm{d}P} = k\sqrt{P}$  or  $\frac{\mathrm{d}P}{\mathrm{d}t} = kP^2$ .

Even though in part (ii) the question asked for candidates to verify that  $P = \left(A + Bt\right)^2$  was a solution to the differential equation many decided instead to solve the differential equation by separating the variables and integrating. It was disappointing how many candidates gave the final answer in this part as  $B = \frac{k}{2}$  even though the question specifically asked for k in terms of B.

#### Question 7

Part (i) was answered extremely well by the vast majority of candidates with nearly all scoring the first three marks for separating the variables and correctly integrating both the x and y terms. The main issue for candidates in this part came in the last two marks as many did not consider the constant of integration. It was disappointing that a small number of candidates either did not state the equation of the curve correctly or did not show sufficient working in verifying this given result.

In part (ii) the vast majority of candidates considered both the correct integral (with correct limits) for the volume of revolution generated by rotating the given curve about the y-axis and went on to integrate correctly. A number of candidates, however, misread the question and instead tried to calculate the volume of revolution generated by rotating the curve about the x-axis. It was also relatively common to see a value of -2 being used as a lower limit on the integral. More worryingly was how many candidates incorrectly re-arranged their expression for  $x^2$  to  $\frac{1}{9}(4y^2-24y)$  which even though it led to a negative volume did not seem to concern many of these candidates. Finally, it was the mention of a rotation of  $180^\circ$  that seemed to concern many candidates and a significant number divided the correct answer of  $\frac{176}{27}\pi$  by 2.

The responses to (iii)(A) were extremely mixed with many candidates making errors in the substitution or expansion of  $9(2\tan\theta)^2 - 4(3(\sec\theta-1))^2 - 24(3(\sec\theta-1))$ . Furthermore, many candidates assumed that this expression was equal to zero and then thought it was mathematically correct to divide through by 36. This highlighted a key misunderstanding even at this level between an expression and an equation. Most candidates correctly used a Pythagorean identity in the simplification of this expression although it was rare for candidates to accurately verify the given equation and therefore score all four marks in this first part.

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Candidates' attempts at (iii)(*B*) were significantly more encouraging but it was slightly disappointing the number of candidates who showed no working in going directly from  $\frac{18 \tan \theta}{12 \sec \theta}$  to  $\frac{3}{2} \sin \theta$ . While

it could easily be argued that a well-prepared candidate could complete the required steps to obtain this final answer without the need to show any working, as the question paper required candidates

to obtain their answer in the form  $k \sin \theta$  the required step of  $\frac{18\left(\frac{\sin \theta}{\cos \theta}\right)}{12\left(\frac{1}{\cos \theta}\right)}$  or equivalent had to be

shown. Those candidates that did not show at least this key step of working failed to understand the demands of this type of question and therefore lost a significant number of marks. Those candidates who had been successful up to this part usually went on to find the gradient of the curve at the point with an x-coordinate of 2 correctly.

#### Question 8

In part (i) most candidates obtained the correct x-coordinate of point B but it was then surprising how many of these candidates incorrectly stated that at this point  $\theta = \pi$  even though this leads to a clearly incorrect value for the y-coordinate. In fact, a number of candidates incorrectly used this value of theta to obtain an answer of y=-1 and then attempted to argue (via symmetry) that y=1.

In part (ii), the majority of candidates correctly calculated the values of R and  $\alpha$  although some lost the first method mark by not including R in the expanded trigonometric statements  $R\cos\alpha=1$  and  $R\sin\alpha=2$ . Some candidates failed to give  $\alpha$  in radians and a small minority stated R as 5 rather than the correct  $\sqrt{5}$ .

Part (iii) was answered well with many candidates correctly deriving the coordinates of points A and C from their answer to part (ii). The main issue seemed to be in not giving answers exactly; in a number of cases the x-coordinates of these two points were given as  $\pm 0.600087...$ 

While the majority of candidates scored the first two marks in part (iv) for correctly stating  $\frac{\mathrm{d}y}{\mathrm{d}x}$  in terms of  $\theta$  (although there were the usual sign errors and errors in differentiating the  $\cos 2\theta$  term) very few candidates correctly substituted the value of  $\theta$ , corresponding to point A, into their expression for  $\frac{\mathrm{d}y}{\mathrm{d}x}$  to obtain the magnitude of either tangent gradient at A. To find  $\beta$  all candidates then had to do was to equate this value to  $\tan\left(\frac{1}{2}\beta\right)$  and solve to obtain the correct answer of 1.90.

#### Paper B

#### Question 1

Both parts of this first question on paper B proved to be surprisingly demanding for many candidates with only a minority correctly stating that in part (i)  $0 < x_0 < 1$  and that in part (ii)  $x_0 = 0$  or  $x_0 = 1$ .

#### Question 2

Even though in part (i) the question specifically asked for candidates to find the equilibrium point by using the algebraic method indicated in the text, many candidates incorrectly used an iterative



# Unit level raw mark and UMS grade boundaries June 2017 series

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## AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

	ematics (MEI)		Max Mark	а	b	С	d	е	ι
4751	01 C1 – MEI Introduction to advanced mathematics (AS)	Raw UMS	72 100	63 80	58 70	53 60	49 50	45 40	(
4752	01 C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	(
	(C2) MEI Matheda for Advanced Mathematica with	UMS	100	80	70	60	50	40	(
1753	01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	(
1753	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	
4753	82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	
	Coursework. Curriou i crivara Coursework Mark	UMS	100	80	70	60	50	40	
4754	01 C4 – MEI Applications of advanced mathematics (A2)	Raw UMS	90 100	67 80	61 70	55 60	49 50	43 40	
4755	61 FP1 – MEI Further concepts for advanced mathematics	Raw	72	57	52	47	42	38	
	(AS)	UMS	100	80	70	60	50	40	
4756	FP2 – MEI Further methods for advanced mathematics	Raw	72	65	58	52	46	40	
	(A2)	UMS	100	80	70	60	50	40	
1757	PP3 – MEI Further applications of advanced mathematics	Raw	72	64	56	48	41	34	
	(A2)	UMS	100	80	70	60	50	40	
1758	01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	_
1758	(DE) MEI Differential Equations with Coursework:  Coursework	Raw	18	15	13	11	9	8	
1758	(DE) MEI Differential Equations with Coursework: Carried	Raw	18	15	13	11	9	8	
	Forward Coursework Mark	UMS	100	80	70	60	50	40	
4761	01 M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	
4762	01 M2 – MEI Mechanics 2 (A2)	UMS Raw	100 72	80 56	70 48	60 41	50 34	40 27	
	01 M2 M21 Modification 2 (1.2)	UMS	100	80	70	60	50	40	
4763	01 M3 – MEI Mechanics 3 (A2)	Raw UMS	72 100	58 80	50 70	43 60	36 50	29 40	
4764	01 M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	
		UMS	100	80	70	60	50	40	
4766	01 S1 – MEI Statistics 1 (AS)	Raw UMS	72 100	61 80	55 70	49 60	43 50	37 40	
4767	01 S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	
		UMS	100	80	70	60	50	40	
4768	01 S3 – MEI Statistics 3 (A2)	Raw UMS	72 100	63 80	57 70	51 60	46 50	41 40	
1769	01 S4 - MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	
4774	04 P4 MEI Parising and a value 4 (40)	UMS	100	80	70	60	50	40	
4771	01 D1 – MEI Decision mathematics 1 (AS)	Raw UMS	72 100	52 80	46 70	41 60	36 50	31 40	
4772	01 D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	
		UMS	100	80	70	60	50	40	
1773	01 DC – MEI Decision mathematics computation (A2)	Raw UMS	72 100	46 80	40 70	34 60	29 50	24 40	
4776	01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	
4776	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	
4776	(NM) MEI Numerical Methods with Coursework: Carried	Raw	18	14	12	10	8	7	
-	<sup>o2</sup> Forward Coursework Mark	UMS	100	80	70	60	50	40	
4777	01 NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	

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		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0
<b>GCE Statis</b>	tics (MEI)								
			Max Mark	а	b	С	d	е	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
GCE Quan	titative Methods (MEI)								
			Max Mark	а	b	С	d	е	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0



# Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Ce	ertificate Mathematics for Engineering									
			Max Mark	a*	а	b	С	d	е	
H860	01 Mathematics for Engineering		This unit	hae no	ontrio	e in lu	na 20	17		
H860	02 Mathematics for Engineering		THIS UTILL	1103 110	CHILLIC	3 111 00	116 20	17		
Lovel 3 Ca	ertificate Mathematical Techniques and Applications for Engine	ore								
Level 5 Ce	er unicate mathematical recliniques and Applications for Engine	CIS	Max Mark	a*	а	b	С	d	е	
H865	01 Component 1	Raw	60	48	42	36	30	24	18	
l aval 2 Ca	autificate Mathematics - Occupationing Pages ving (MEI) (CO Defe									
Level 3 Ce	ertificate Mathematics - Quantitative Reasoning (MEI) (GQ Refo	rm)	Max Mark	а	b	С	d	е	u	
H866	01 Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0	•
H866	02 Critical maths	Raw	60*	48	42	36	30	24	0	
	*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0	_
Level 3 Ce	ertificate Mathematics - Quantitive Problem Solving (MEI) (GQ F	Reform)								
			Max Mark	а	b	С	d	е	u	
H867	01 Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0	
H867	02 Statistical problem solving	Raw	60*	41	36	31	27	23	0	
	*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0	
Advanced	Free Standing Mathematics Qualification (FSMQ)									
Auvanceu	Tree standing mathematics additionation (1 only)		Max Mark	а	b	С	d	е	u	
6993	01 Additional Mathematics	Raw	100	72	63	55	47	39	0	
Intermedia	ate Free Standing Mathematics Qualification (FSMQ)		May May!				al			
6090	01 Foundations of Advanced Mathematics (MFI)	Pov:	Max Mark	a 25	<b>b</b>	C	d	<b>e</b>	u	ı
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0	

Published: 16 August 2017 Version 1.0 1





# Friday 23 June 2017 – Morning

# **A2 GCE MATHEMATICS (MEI)**

**4754/01B** Applications of Advanced Mathematics (C4) Paper B: Comprehension

#### **QUESTION PAPER**

Candidates answer on the Question Paper.

#### **OCR** supplied materials:

- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

#### Other materials required:

Scientific or graphical calculator

**Duration:** Up to 1 hour



Candidate forename				Candidate surname			
Centre numb	er			Candidate nu	umber		

### **INSTRUCTIONS TO CANDIDATES**

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer all the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do not write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are not required to hand in these notes with your question paper.
- You are advised that an answer may receive no marks unless you show sufficient detail
  of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 18.
- This document consists of 8 pages. Any blank pages are indicated.

1 S	tate the	set of values	of $x_0$ f	or which	the iteration
-----	----------	---------------	------------	----------	---------------

$$x_{n+1} = 2.5x_n(1 - x_n)$$

- (i) converges to a single non-zero number, [1]
- (ii) has all terms from  $x_1$  onwards equal to zero. [1]

1(i)	
1(ii)	

2	(i)	Use the algebraic	method inc	dicated in lin	nes 68 to 70 to	find the eq	uilibrium p	oint of the	iteration
_	11/	Obe the digeorale	miculou mic	arcated iii iiii		J IIII a are eq	dillollalli p	Omit of the	itti atio.

$$x_{n+1} = 1.6x_n(1-x_n).$$
 [2]

(ii) Show that the iteration

$$x_{n+1} = x_n^2 + 2$$

does not have any points of equilibrium.

[2]

2(i)	
2(ii)	

3 One of the assumptions for the model used for the population of squirrels in the text was that there are no predators.

An alternative model is proposed in which predators kill a fixed number of squirrels each year.

An iterative equation for this model is given by

$$x_{n+1} = kx_n(1-x_n) - 0.25.$$

In the table below  $x_0$  is taken to be 0.55 and four different values are considered for k.

- (i) Complete as many of the empty cells as you need to in order to establish the outcomes for these values of *k*.
- (ii) Comment on what the table tells you for each of the four values of k. [6]

		$x_{n+1} = kx_n(1)$	$(1-x_n) - 0.25$	
	<i>k</i> = 2	k=3	k = 4	<i>k</i> = 5
$x_0$	0.55	0.55	0.55	0.55
$x_1$	0.245	0.4925	0.74	0.9875
$x_2$				
$x_3$				
$x_4$				
$x_5$				
$x_6$				
$x_7$				
$x_8$				
$x_9$				
<i>x</i> <sub>10</sub>				

3(ii)	

4 (i) Table 3 gives the first four points of bifurcation of the iteration

$$x_{n+1} = kx_n(1 - x_n).$$

Feigenbaum's Constant is 4.6692 correct to 5 significant figures. Using this value for the ratio of the interval lengths, estimate the values of k for the next two points of bifurcation. [3]

(ii) (A) Find, S, the sum to infinity of the geometric series

$$1 + \frac{1}{4.6692} + \left(\frac{1}{4.6692}\right)^2 + \left(\frac{1}{4.6692}\right)^3 + \dots$$

[2]

(B) Using certain figures from Table 3, a value of k is estimated to be

$$k = 3.5644 + 0.0203 \times S$$
.

State what happens at this value of k.

[1]

<b>4(i)</b>	

4(ii)(A)	
4(ii)( <i>B</i> )	

# END OF QUESTION PAPER

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# Friday 23 June 2017 - Morning

# **A2 GCE MATHEMATICS (MEI)**

**4754/01B** Applications of Advanced Mathematics (C4) Paper B: Comprehension **INSERT** 

**Duration:** Up to 1 hour



## **INFORMATION FOR CANDIDATES**

- This Insert contains the text for use with the questions.
- This document consists of 8 pages. Any blank pages are indicated.

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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# Feigenbaum's Constant

## A population model

A remote island has a population of squirrels. There are no predators so that the population is limited only by the supply of food and death from natural causes.

How would the number of squirrels be expected to change from year to year?

5

10

A simple model involves the following variables.

- Time is measured in years and the year number is denoted by n. In this article, year n is also described as 'this year' and year n+1 as 'next year'.
- The size of the squirrel population is given as the proportion of the maximum possible population; the population is given at the start of each year. The population in year n is denoted by  $x_n$  where  $0 \le x_n \le 1$ . Similarly, in year n+1 the population is  $x_{n+1}$ .
- A parameter k is a measure of the reproductivity of the squirrels, and so determines the rate of growth of their population.

The model involves the following assumptions.

- The number of squirrels next year,  $x_{n+1}$ , is jointly proportional to the number this year,  $x_n$ , and to the quantity  $(1-x_n)$  which represents the food available.
- The parameter k is the constant of proportionality.

The model can be expressed by the iterative equation

$$x_{n+1} = kx_n(1-x_n).$$

This is a general model for a population in a restricted environment without predators. The animals do not have to be squirrels.

#### Different values of k

To investigate this model, it is first necessary to choose values for k and for the initial population,  $x_0$ . Table 1 gives the first ten values of  $x_n$  for four particular values of k and with  $x_0 = 0.5$ . The numbers given in the table have been truncated; many more figures were used in calculating them.

25

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		$x_{n+1} = kx$	$x_n(1-x_n)$	
	k = 0.6	k = 1.5	k = 3.3	k = 4.5
$x_0$	0.5	0.5	0.5	0.5
$x_1$	0.15	0.375	0.825	1.125
$x_2$	0.0765	0.3515	0.4764	-0.6328
$x_3$	0.0423	0.3419	0.8231	-4.649
$x_4$	0.0243	0.3375	0.4803	-118.2
$x_5$	0.0142	0.3354	0.8237	-
$x_6$	0.0084	0.3343	0.4791	_
$x_7$	0.0050	0.3338	0.8235	_
$x_8$	0.0029	0.3335	0.4795	-
$x_9$	0.0017	0.3334	0.8236	_
<i>x</i> <sub>10</sub>	0.0010	0.3333	0.4794	-

**Table 1** 40

The different values of *k* used in Table 1 lead to four different outcomes for the population in this model.

- When k = 0.6,  $x_n \to 0$ . Eventually the population dies out.
- When k = 1.5,  $x_n \to 0.3333... = \frac{1}{3}$ . The population attains a stable equilibrium level.
- When k = 3.3, the population alternates between a high value of 0.823... and a low value of 0.479... in successive years.

45

• When k = 4.5, the population appears to become negative and so to have died out. In the first year the population exceeds the limit of 1 and so the model has broken down.

In each case in Table 1 the value of  $x_0$  was taken to be 0.5, the middle of the possible values of  $x_0$ . If you try other starting values you will find that the final outcomes are the same for any values of  $x_0$  between, but not including, 0 and 1.

50

Overall the model suggests that having too few or too many young can both be fatal for the population.

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#### Possible outcomes

This iteration can be used as a population model, but it can also be thought of as a mathematical iteration in its own right with an interesting variety of possible outcomes.

At this stage it is helpful to extend the notation used to include the letter x. This denotes the value, or values, of  $x_n$  as n tends to infinity.

55

In Fig. 2, these limiting values, x, are plotted for values of k between 0 and 3.6. For larger values of k there are no limiting values. It is assumed that  $0 < x_0 < 1$ .

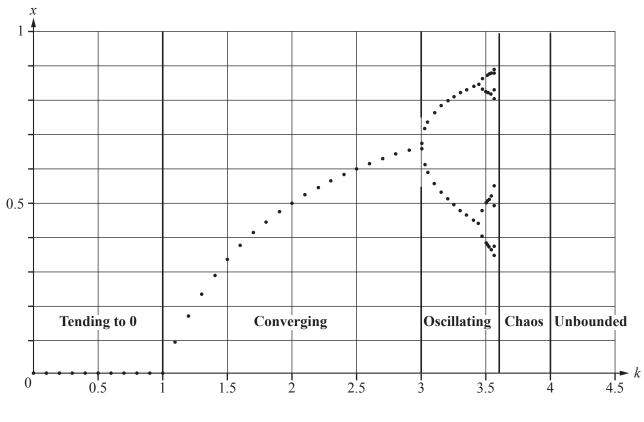


Fig. 2

Fig. 2 shows that, for this iteration, five different types of outcome are possible according to the value of k. These are described below.

60

#### Tending to zero

For  $0 \le k \le 1$ , the iteration converges to zero.

#### Converging to a single non-zero value

For  $1 \le k \le 3$ , the iteration converges to a non-zero value between 0 and 1.

65

An example of this is given in Table 1 for k = 1.5. The iteration is found to converge to  $\frac{1}{3}$ . This value may be described as an *equilibrium point*.

It can be found algebraically. Denoting it by x gives the equation x = 1.5x(1-x)

which has a solution of x = 0 or  $\frac{1}{3}$ .

70

### **Oscillating**

Table 1 shows that for k = 3.3,  $x_n$  oscillates between two values. There is a range of values of k for which this occurs.

• For k = 2.99,  $x_n$  converges slowly to 0.6655... . At k = 3, it starts to oscillate. After 5000 iterations the low value is 0.6633... and the high value is 0.6699... . This is a cycle of length 2.

75

Thus it is found, for example by experiment using a spreadsheet, that the smallest value of *k* for which the iteration oscillates is 3. Such a value of *k* where the iteration splits is called a *point of bifurcation*.

However, k = 3 is not the only point of bifurcation. At k = 3.449... there is a further point of bifurcation at which the cycle of length 2 becomes a cycle of length 4.

• For k = 3.5, the four values of x are 0.3828..., 0.8269..., 0.5008... and 0.8749...

80

Another point of bifurcation occurs at k = 3.544... At this point the length of the cycle goes up to 8. Further points of bifurcation give cycles of length 16, then 32, then 64 and so on.

#### Chaos

The pattern of cycles of increasing length does not continue indefinitely as k increases. For larger values of k, for example k = 3.8, the outcomes have no pattern, and so the situation is described as *chaos*. It is very difficult to distinguish between chaos and a long cycle; a sophisticated computer program is required.

85

A feature of chaos is that the iteration remains bounded. The values of  $x_n$  are always between 0 and 1.

### Unbounded outcomes

For values of  $k \ge 4$ , the outcomes cease to be bounded. An example of this occurs in the final column of Table 1, where the value of k is 4.5.

90

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#### Feigenbaum's Constant

To summarise, from k = 1 to 3 the iteration  $x_{n+1} = kx_n(1-x_n)$  converges to a single non-zero value. There is then a point of bifurcation and this is followed by further points of bifurcation. It is evident from Fig. 2 that the intervals between successive points of bifurcation become progressively shorter.

95

Information about these intervals is given in Table 3. The numbers in this table have been rounded to 4 decimal places.

Cycle length	Vele length         Boundary           1         1           2         3           4         3.4495           8         3.5441           16         3.5644	values of <i>k</i>	Interval	Previous interval This interval
1	1	3	2	_
2	3	3.4495	0.4495	4.4494
4	3.4495	3.5441	0.0946	4.7516
8	3.5441	3.5644	0.0203	4.6601
16	3.5644			

100

Table 3

The right hand column gives the ratio by which the length of this interval decreases with successive cycles. The three values of this ratio in Table 3 are close together.

105

In the 1970s, Mitchell Feigenbaum started to investigate this ratio. He discovered that similar patterns of bifurcation are found with many other iterations; examples include  $x_{n+1} = x_n^2 + k$  and  $x_{n+1} = k \sin(\pi x_n)$ .

He also discovered that the ratio tends to a definite limit and that this has the same value for all iterations that show this pattern of bifurcation.

110

His work was conducted to a very high level of accuracy and covered many more cycles than the small number considered here. The limited power of computers in those days meant that it was an enormous undertaking.

Feigenbaum was immediately convinced of the importance of his discovery.

'I called my parents that evening and told them I had discovered something truly remarkable, that, when I had understood it, would make me a famous man.'

115

He is indeed now a famous man and the ratio he discovered is called Feigenbaum's Constant. It has now been found to over 1000 figures; the first ten of these are 4.669 201 609.

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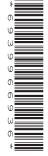


# Friday 23 June 2017 - Morning

# **A2 GCE MATHEMATICS (MEI)**

**4754/01** Applications of Advanced Mathematics (C4)

**INSTRUCTIONS** 



The examination is in two parts:

Paper A (1 hour 30 minutes) Paper B (up to 1 hour)

Supervisors are requested to ensure that Paper B is not issued until Paper A has been collected in from the candidates.

Centres may, if they wish, grant a supervised break between the two parts of this examination.

Paper B should not be attached to the corresponding Paper A script. For Paper A only the candidates' Printed Answer Books, in the same order as the attendance register, should be sent for marking; the Question Paper should be retained in the centre or recycled. For Paper B only the Question Papers, on which the candidates have written their answers, should be sent for marking; the Insert should be retained in the centre or recycled. Any additional sheets used must be carefully attached to the correct paper.

For Paper B (Comprehension) only.

A standard English dictionary is allowed for the comprehension.

(Dictionaries to be used in the examination must be thoroughly checked before the examination.) Full regulations are in the JCQ Regulations and Guidance booklet.

This notice must be on the invigilator's desk at all times during the morning of Friday 23 June 2017.



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# **GCE**

# **Mathematics (MEI)**

Unit 4754B: Applications of Advanced Mathematics: Paper B

Advanced GCE

Mark Scheme for June 2017

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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# **Annotations and abbreviations**

Annotation in scoris	Meaning
✓ and 💥	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
MO M1	Method mark awarded 0, 1
A0 A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations	Meaning
in mark scheme	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
WWW	Without wrong working

## Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

#### М

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

Ε

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

- If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
- NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.
  - Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Qu		Answer	Marks	Guidance
1.	(i)	(All $x_0$ for which) $0 < x_0 < 1$	B1	Condone as separate inequalities – allow $x$ , $x_n$ , etc. for $x_0$ Accept $(0, 1)$ (i.e. correct set-notation) – must be strict inequalities
			[1]	
	(ii)	$x_0 = 0 \text{ or } 1$	B1	Both required - allow $x, x_n$ , etc. for $x_0$ - condone 0, 1 stated without $x_0$ (0 and 1 must not appear as part of a range of values)
			[1]	

2.	(i)	x = 1.6x(1-x)	M1	For correctly replacing $x_{n+1}$ and $x_n$ with $x$ - must be using 1.6
		x(1.6x - 0.6) = 0		
		$(x = 0 \text{ or}) \ x = \frac{0.6}{1.6} = 0.375 \text{ or } \frac{3}{8}$	<b>A1</b>	Using an iterative approach or stating the correct answer with no working - M0 A0
				Accept as a minimum the correct equation stated in terms of <i>x</i> only followed by the correct answer for both marks
			[2]	
2.	(ii)	$x^2 - x + 2 = 0$		
		Discriminant = $(-1)^2 - 4 \times 1 \times 2 = -7$ or $x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(2)}}{2}$	M1	Re-arranging <b>and</b> correct attempt at either the discriminant or solving using the correct formula/completing the square for their 3-term quadratic equation. Allow consideration of $\sqrt{b^2-4ac}$ with correct substitution of their values only
		so no (real) roots (oe)	A1	Correct working with a correct conclusion (so if discriminant stated must be -7 but may be awarded for $1-4(2) \Rightarrow$ no (real) roots/no points of equilibrium)  No marks for non-algebraic approach
			[2]	

# PLEASE NOTE THAT THE MARKS FOR BOTH 3(i) and 3(ii) MUST BE AWARDED TOGETHER (MAXIMUM OF 6)

3.	<b>(i)</b>			$x_{n+1} = kx_n  \big($	$(1-x_n)-0.25$				There may be some variation in the columns for $k = 3$ and $k = 4$ . Some calculators will round to the
			<i>k</i> = 2	<i>k</i> = 3	k = 4	<i>k</i> = 5			final values earlier than others. In general answers must be given to at least 2dp
		$x_0$	0.55	0.55	0.55	0.55			Allow correct truncated or rounded answers
		$x_1$	0.245	0.4925	0.74	0.9875			
		$x_2$	0.1199	0.4998	0.5196	-0.1882			One mark for each column taken far enough
		$x_3$	-0.0388	0.4999	0.7484			B1	$k = 2$ correct to at least $x_3$
		$X_4$		0.5	0.5030			B1	$k = 3$ correct to at least $x_3$ (accept 0.5)
		$x_5$			0.7499				
		$x_6$			0.5000			B1	$k = 4$ correct to at least $x_6$ (accept 0.5)
		$x_7$			0.7499			B1	$k = 5$ correct to at least $x_2$
		$\mathcal{X}_8$			0.5000				$\kappa = 3$ correct to at least $\kappa_2$
		$X_9$			0.75				isw after reaching the values for each column as
		$\mathcal{X}_{10}$			0.5				stated above
							_		

3.	( <b>ii</b> )		B2	All 4 correct Allow B1 for 2 correct
		For $k = 2$ it is <b>unbounded</b> or ' <b>dying out</b> '		Allow equivalent to 'dying out' provided unambiguous
		For $k = 3$ it <b>converges</b> (to $x = 0.5$ ) or <b>stable</b> or <b>equilibrium</b>		Must be one of these three words
		For $k = 4$ it <b>oscillates</b> (between $x = 0.5$ and $x = 0.75$ ) or <b>alternates</b> but not <b>fluctuates</b>		Must be one of these two words
		For $k = 5$ , it is <b>unbounded</b> or ' <b>dying out</b> '		Allow equivalent to 'dying out' provided unambiguous
			[6]	

4.	(i)	$3.5644 + \frac{0.0203}{4.6692}$	В1	Correct method - allow a slip in one value (e.g. 4.692) only – either correct value for <i>k</i> implies this mark
		= 3.5687	B1	Accept 3.569 (3.5687476) – no ft from incorrect values – accept if given as the upper bound of an interval
		$3.5687 + 0.0203 \times \left(\frac{1}{4.6692}\right)^2 = 3.5696 \text{ to 5 sf}$	B1	Accept 3.5697 or 3.570 but not 3.57 – no ft from incorrect values – accept if given as an upper bound
			[3]	
4	(ii)(A)	$S = \frac{1}{1 - \frac{1}{4.6692}}$ $S = 1.2725$	M1	Use of the correct formula for the sum to infinity of a GP – allow a slip in the value of $r$ only (e.g. $4.692$ ) – allow $r$ stated to 2dp or better $(r = 0.21416945)$ Accept 1.273 not 1.27 (1.2725389)
				M0 A0 if formula not used
			[2]	
4	(ii)( <i>B</i> )	It is an estimate of a value of k for which chaos is occurring	B1	Accept any mention of 'chaos'
			[1]	

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# Unit level raw mark and UMS grade boundaries June 2017 series

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## AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

	ematics (MEI)		Max Mark	а	b	С	d	е	ı
4751	01 C1 – MEI Introduction to advanced mathematics (AS)	Raw UMS	72 100	63 80	58 70	53 60	49 50	45 40	(
4752	01 C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	(
	(C2) MEI Mathada for Advanced Mathamatica with	UMS	100	80	70	60	50	40	(
1753	01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	(
1753	02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	
4753	82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	
	Coursework. Carried Forward Coursework Mark	UMS	100	80	70	60	50	40	
4754	01 C4 – MEI Applications of advanced mathematics (A2)	Raw UMS	90 100	67 80	61 70	55 60	49 50	43 40	
1755	FP1 – MEI Further concepts for advanced mathematics	Raw	72	57	52	47	42	38	
	(AS)	UMS	100	80	70	60	50	40	
4756	FP2 – MEI Further methods for advanced mathematics	Raw	72	65	58	52	46	40	
	(A2)	UMS	100	80	70	60	50	40	
1757	FP3 – MEI Further applications of advanced mathematics	Raw	72	64	56	48	41	34	
	(A2)	UMS	100	80	70	60	50	40	
1758	01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	
1758	02 (DE) MEI Differential Equations with Coursework:	Raw	18	15	13	11	9	8	
1758	Coursework  (DE) MEI Differential Equations with Coursework: Carried	Raw	18	15	13	11	9	8	
., 00	Forward Coursework Mark	UMS	100	80	70	60	50	40	
1761	01 M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	
4762	O4 MO MEI Machanica 2 (A2)	UMS	100 72	80	70	60	50 34	40	
+/62	01 M2 – MEI Mechanics 2 (A2)	Raw UMS	100	56 80	48 70	41 60	50	27 40	
4763	01 M3 – MEI Mechanics 3 (A2)	Raw UMS	72 100	58	50 70	43	36 50	29 40	
4764	01 M4 – MEI Mechanics 4 (A2)	Raw	100 72	80 53	45	60 38	31	24	
1704	or with with worldings 4 (NZ)	UMS	100	80	70	60	50	40	
4766	01 S1 – MEI Statistics 1 (AS)	Raw	72	61	55 70	49	43	37	
4767	01 S2 – MEI Statistics 2 (A2)	UMS Raw	100 72	80 56	70 50	60 45	50 40	40 35	_
77 07	or oz wier otatistics z (viz)	UMS	100	80	70	60	50	40	
1768	01 S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	
1700	04 04 MELOCITIC 4 (A0)	UMS	100	80	70	60	50	40	
1769	01 S4 – MEI Statistics 4 (A2)	Raw UMS	72 100	56 80	49 70	42 60	35 50	28 40	
771	01 D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	-
	0. 2 <u></u>	UMS	100	80	70	60	50	40	
772	01 D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	
1770	Of DO MEI Desiring and the ordinary (AC)	UMS	100	80	70	60	50	40	
773	01 DC – MEI Decision mathematics computation (A2)	Raw UMS	72 100	46 80	40 70	34 60	29 50	24 40	
776	01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	
1776	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	
1776	(NM) MEI Numerical Methods with Coursework: Carried	Raw	18	14	12	10	8	7	
	Forward Coursework Mark	UMS	100	80	70	60	50	40	
4777	01 NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	

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# 4754 Applications of Advanced Mathematics (C4)

#### **General Comments:**

On the whole candidates found Paper A this year slightly less demanding than last year and the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Paper B, the comprehension, was well understood and most candidates scored good marks here.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Questions 1, 2, 4, 7(ii) and 7(iii))
- Failure to include a constant of integration (Question 7(i))
- Failure to give clear descriptions in the comprehension paper (Questions 2 and 3)
- Inappropriate accuracy, for example in Question 3(ii), candidates either gave insufficient accuracy (answers to 1 decimal place) or they gave too much accuracy (answers to 3 or more decimal places). Candidates are reminded to give answers to 1 decimal place for questions involving trigonometry (Questions 2(ii) and 5)
- Failure to give exact answers when required (Question 7)
- Failure to give sufficient detail when verifying given results (Questions 6(ii), 7(i) and 7(iii))

Quite a number of candidates did not attempt some parts but there did not appear to be a shortage of time for either Paper.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and are sent separately for marking.

#### **Comments on Individual Questions:**

Paper A

Question 1

Part (i) was answered extremely well with the vast majority of candidates correctly expressing  $\frac{5-x}{(2-x)(1+x)}$  in partial fractions.

In part (ii) most candidates used their answer to part (i) in their attempt to find the binomial expansion of  $\frac{5-x}{(2-x)(1+x)}$  although some candidates did (with varying degrees of success) attempt to expand  $(5-x)(2-x)^{-1}(1+x)^{-1}$  directly. Whilst the majority of candidates correctly dealt with the expansion of  $\frac{2}{1+x}$  (and so scored at least two marks in this part) it was surprising how many

candidates (at this level) struggled in re-writing  $\frac{1}{2-x}$  as  $\frac{1}{2}\left(1-\frac{1}{2}x\right)^{-1}$ . In some cases it was clear

that candidates either did not realise or even recognise that the 2 inside the bracket had to be removed before this term could be binomially expanded. Those candidates who expanded both terms correctly usually went on to score full marks.

Question 2

Part (i) was answered extremely well with the vast majority of candidates correctly substituting the parametric form of the equation of the line into the equation of the plane to find the value of the parameter at the point of intersection (which if done correctly gave  $\lambda = -3$ ). This value was then substituted back into the equation of the line to find the coordinates of the point of intersection. Although many candidates gave their answer as a position vector (rather than as coordinates) this was condoned by examiners.

The main stumbling block in part (ii) was not the application of the scalar product to find the angle between the line and the normal to the plane but it was in the choosing of the correct two direction

vectors; in many cases candidates incorrectly used either 
$$\begin{pmatrix} -7\\9\\-2 \end{pmatrix}$$
, or even more surprisingly  $\begin{pmatrix} -1\\0\\1 \end{pmatrix}$ 

It was pleasing to note that in most cases, where candidates found the correct obtuse angle of 111.5°, they then correctly subtracted this value from 180° to obtain the correct acute angle.

#### Question 3

The first mark in part (i) was awarded to the vast majority of candidates for correctly stating that  $T_4$  was less than  $T_2$  although some candidates did not make it explicitly clear which value of the two was the least. Candidates found the second mark a lot harder to come by as it was not sufficient to simply state that the approximations given by the trapezium rule were an over-estimate. Candidates needed to make it clear that these approximations were an over-estimate because the tops of the trapezia are above the curve which would then, in turn, mean that the error (in using a trapezium rule approximation for the value of the integral) would become less when the number of strips increases.

Part (ii) was answered extremely well with the vast majority of candidates giving the correct answer of 3.25. When errors occurred it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the *x* values or to not give the answer to the required 3 significant figures.

### Question 4

On the whole, candidates made an impressive start to this unstructured vector question and the majority of candidates correctly achieved  $\sqrt{a^2+b^2+25}=\sqrt{27}$  and a-7b-10=0. It was therefore extremely disappointing that at this level so many candidates failed to solve this pair of simultaneous equations accurately. Too many candidates simplified  $\sqrt{a^2+b^2+25}=\sqrt{27}$  to a+b+5=27 or re-wrote a-7b-10=0 as  $a^2-49b^2-100=0$  and so therefore did not obtain a three-term quadratic equation in either a or b. Of those that did achieve a correct equation, most notably  $(7b+10)^2+b^2=2$ , it was disappointing how many then expanded this as

 $49b^2 + 100 + b^2 = 2$  and still arrived at a real solution for the value of b from the equation  $b^2 = -\frac{49}{25}$ .

For those candidates who did expand correctly most achieved the correct values for both a and b.

#### Question 5

It was pleasing to note that most candidates used the correct double angle formulae for  $\tan 2\theta$  to obtain a correct equation in terms of  $\tan \theta$ . However, some candidates over complicated the problem by re-writing tan in terms of sin and cos and in these cases it was extremely rare for candidates to make any real significant progress. Of those that correctly re-arranged

$$4\tan\theta \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=1$$
 to  $\tan^2\theta=\frac{1}{9}$  it was disappointing that so many candidates then only considered the solutions of the equation  $\tan\theta=\frac{1}{3}$  and ignored any possible solutions that would have come from the equation  $\tan\theta=-\frac{1}{3}$ .

#### Question 6

In part (i) most candidates correctly wrote down the differential equation relating P, the time t, and the constant k. The most common errors in this part were those candidates who wrote  $\frac{\mathrm{d}t}{\mathrm{d}P} = k\sqrt{P}$  or  $\frac{\mathrm{d}P}{\mathrm{d}t} = kP^2$ .

Even though in part (ii) the question asked for candidates to verify that  $P = \left(A + Bt\right)^2$  was a solution to the differential equation many decided instead to solve the differential equation by separating the variables and integrating. It was disappointing how many candidates gave the final answer in this part as  $B = \frac{k}{2}$  even though the question specifically asked for k in terms of B.

#### Question 7

Part (i) was answered extremely well by the vast majority of candidates with nearly all scoring the first three marks for separating the variables and correctly integrating both the x and y terms. The main issue for candidates in this part came in the last two marks as many did not consider the constant of integration. It was disappointing that a small number of candidates either did not state the equation of the curve correctly or did not show sufficient working in verifying this given result.

In part (ii) the vast majority of candidates considered both the correct integral (with correct limits) for the volume of revolution generated by rotating the given curve about the y-axis and went on to integrate correctly. A number of candidates, however, misread the question and instead tried to calculate the volume of revolution generated by rotating the curve about the x-axis. It was also relatively common to see a value of -2 being used as a lower limit on the integral. More worryingly was how many candidates incorrectly re-arranged their expression for  $x^2$  to  $\frac{1}{9}(4y^2-24y)$  which even though it led to a negative volume did not seem to concern many of these candidates. Finally, it was the mention of a rotation of  $180^\circ$  that seemed to concern many candidates and a significant number divided the correct answer of  $\frac{176}{27}\pi$  by 2.

The responses to (iii)(A) were extremely mixed with many candidates making errors in the substitution or expansion of  $9(2\tan\theta)^2 - 4(3(\sec\theta-1))^2 - 24(3(\sec\theta-1))$ . Furthermore, many candidates assumed that this expression was equal to zero and then thought it was mathematically correct to divide through by 36. This highlighted a key misunderstanding even at this level between an expression and an equation. Most candidates correctly used a Pythagorean identity in the simplification of this expression although it was rare for candidates to accurately verify the given equation and therefore score all four marks in this first part.

## OCR Report to Centres - June 2017

Candidates' attempts at (iii)(*B*) were significantly more encouraging but it was slightly disappointing the number of candidates who showed no working in going directly from  $\frac{18 \tan \theta}{12 \sec \theta}$  to  $\frac{3}{2} \sin \theta$ . While

it could easily be argued that a well-prepared candidate could complete the required steps to obtain this final answer without the need to show any working, as the question paper required candidates

to obtain their answer in the form  $k \sin \theta$  the required step of  $\frac{18\left(\frac{\sin \theta}{\cos \theta}\right)}{12\left(\frac{1}{\cos \theta}\right)}$  or equivalent had to be

shown. Those candidates that did not show at least this key step of working failed to understand the demands of this type of question and therefore lost a significant number of marks. Those candidates who had been successful up to this part usually went on to find the gradient of the curve at the point with an x-coordinate of 2 correctly.

#### Question 8

In part (i) most candidates obtained the correct x-coordinate of point B but it was then surprising how many of these candidates incorrectly stated that at this point  $\theta = \pi$  even though this leads to a clearly incorrect value for the y-coordinate. In fact, a number of candidates incorrectly used this value of theta to obtain an answer of y=-1 and then attempted to argue (via symmetry) that y=1.

In part (ii), the majority of candidates correctly calculated the values of R and  $\alpha$  although some lost the first method mark by not including R in the expanded trigonometric statements  $R\cos\alpha=1$  and  $R\sin\alpha=2$ . Some candidates failed to give  $\alpha$  in radians and a small minority stated R as 5 rather than the correct  $\sqrt{5}$ .

Part (iii) was answered well with many candidates correctly deriving the coordinates of points A and C from their answer to part (ii). The main issue seemed to be in not giving answers exactly; in a number of cases the x-coordinates of these two points were given as  $\pm 0.600087...$ 

While the majority of candidates scored the first two marks in part (iv) for correctly stating  $\frac{\mathrm{d}y}{\mathrm{d}x}$  in terms of  $\theta$  (although there were the usual sign errors and errors in differentiating the  $\cos 2\theta$  term) very few candidates correctly substituted the value of  $\theta$ , corresponding to point A, into their expression for  $\frac{\mathrm{d}y}{\mathrm{d}x}$  to obtain the magnitude of either tangent gradient at A. To find  $\beta$  all candidates then had to do was to equate this value to  $\tan\left(\frac{1}{2}\beta\right)$  and solve to obtain the correct answer of 1.90.

#### Paper B

#### Question 1

Both parts of this first question on paper B proved to be surprisingly demanding for many candidates with only a minority correctly stating that in part (i)  $0 < x_0 < 1$  and that in part (ii)  $x_0 = 0$  or  $x_0 = 1$ .

#### Question 2

Even though in part (i) the question specifically asked for candidates to find the equilibrium point by using the algebraic method indicated in the text, many candidates incorrectly used an iterative

method. For those candidates that stated x = 1.6x(1-x) most went on to solve this equation correctly.

In part (ii) whilst many candidates correctly stated that at the point of equilibrium  $x = x^2 + 2$  many did not give a mathematically sound reason for why this equation does not have any real solutions. It was insufficient to use an iterative approach and it was simply incorrect to state that the quadratic equation does not factorise. However, many candidates did re-arrange the equation correctly and then proceeded to show that the discriminant of the resulting three-term quadratic was negative.

#### Question 3

This question was answered extremely well with many candidates scoring all four marks in part (i). The vast majority of candidates scored the three marks for filling in the table correctly for k = 2,3 and 5 but for the case when k = 4 a number of candidates did not complete a sufficient number of empty cells to clearly establish the outcome for this value.

Most candidates in part (ii) commented extremely well on what the values in the table implied about the four values of k; the main error being in the use of incorrect phrases or terminology (e.g. using 'fluctuating' to describe the behaviour when k = 4).

#### Question 4

Candidates seemed to be evenly split in part (i) in either making no real progress when attempting to find the values of k (for the next two points of bifurcation) or they correctly found both. Where issues occurred it was mainly due to accuracy.

In part (ii)(A) it was expected that candidates would use the sum to infinity formula for a geometric series and not use the less mathematically sound approach of simply adding up the first few terms on their calculator. Furthermore, it was surprising in this part how many candidates either struggled

with finding the common ratio (many claiming that r = 4.6692 rather than the correct  $r = \frac{1}{4.6692}$ )

or gave a final answer to a degree of accuracy which was not appropriate to the given context.

Finally, in part (ii)(B), while many candidates left this part blank or simply worked out the value of k it was encouraging that many candidates correctly stated that at this value chaos would be occurring.



		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0
<b>GCE Statis</b>	stics (MEI)								
			Max Mark	а	b	С	d	е	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
	. ,	UMS	100	80	70	60	50	40	0
GCE Quan	titative Methods (MEI)								
002 4,00			Max Mark	а	b	С	d	е	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0



# Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Ce	ertificate Mathematics for Engineering									
			Max Mark	a*	а	b	С	d	е	
H860	01 Mathematics for Engineering		This unit	hae no	ontrio	e in lu	na 20	17		
H860	02 Mathematics for Engineering		THIS UTILL	1103 110	CHILLIC	3 111 00	116 20	17		
Lovel 3 Ca	ertificate Mathematical Techniques and Applications for Engine	ore								
Level 5 Ce	er unicate mathematical recliniques and Applications for Engine	CIS	Max Mark	a*	а	b	С	d	е	
H865	01 Component 1	Raw	60	48	42	36	30	24	18	
l aval 2 Ca	autificate Mathematics - Occupationing Pages ving (MEI) (CO Defe									
Level 3 Ce	ertificate Mathematics - Quantitative Reasoning (MEI) (GQ Refo	rm)	Max Mark	а	b	С	d	е	u	
H866	01 Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0	•
H866	02 Critical maths	Raw	60*	48	42	36	30	24	0	
	*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0	_
Level 3 Ce	ertificate Mathematics - Quantitive Problem Solving (MEI) (GQ F	Reform)								
			Max Mark	а	b	С	d	е	u	
H867	01 Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0	
H867	02 Statistical problem solving	Raw	60*	41	36	31	27	23	0	
	*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0	
Advanced	Free Standing Mathematics Qualification (FSMQ)									
Auvanceu	Tree standing mathematics additionation (1 only)		Max Mark	а	b	С	d	е	u	
6993	01 Additional Mathematics	Raw	100	72	63	55	47	39	0	
Intermedia	ate Free Standing Mathematics Qualification (FSMQ)		May May!				al			
6090	01 Foundations of Advanced Mathematics (MFI)	Pov:	Max Mark	a 25	<b>b</b>	C	d	<b>e</b>	u	ı
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0	

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